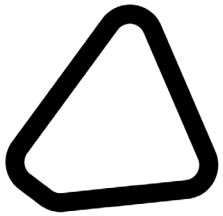


convex risk

The Cost of Goods Sold

Stephen Mildenhall
John Major
CARe Meeting, June 9, 2021

The opinions expressed in this presentation are not necessarily those of the presenters.

Live Poll

Which best describes your employer?

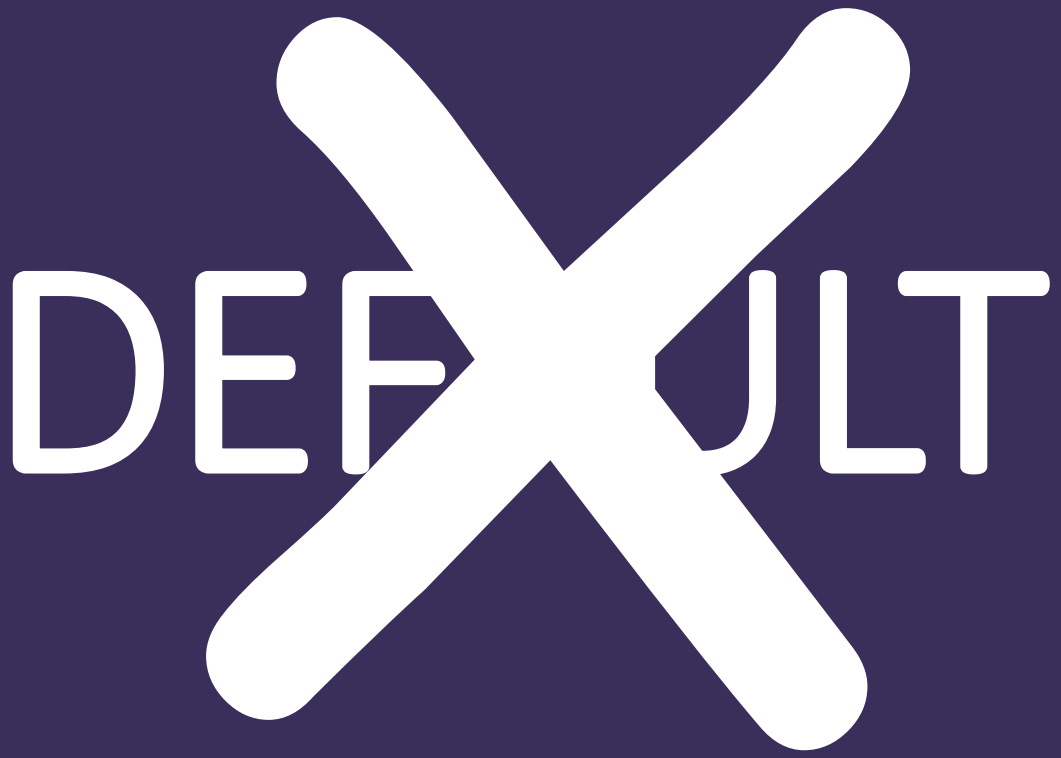
Answers

- Insurer
- Reinsurer
- Broker
- Consultant
- Regulator, rating agency, other

Live Poll Answers

Ten Reasons to Love Distortions

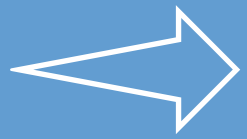
$$\{\text{risk}\} \times \{\text{capital structure}\} \rightarrow \text{price}$$

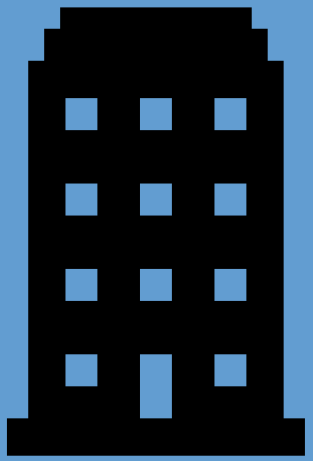


The Cast

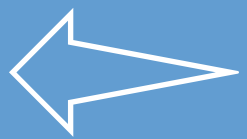
Regulator

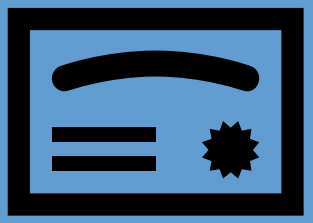
Risks





Ins Co.





Capital Markets

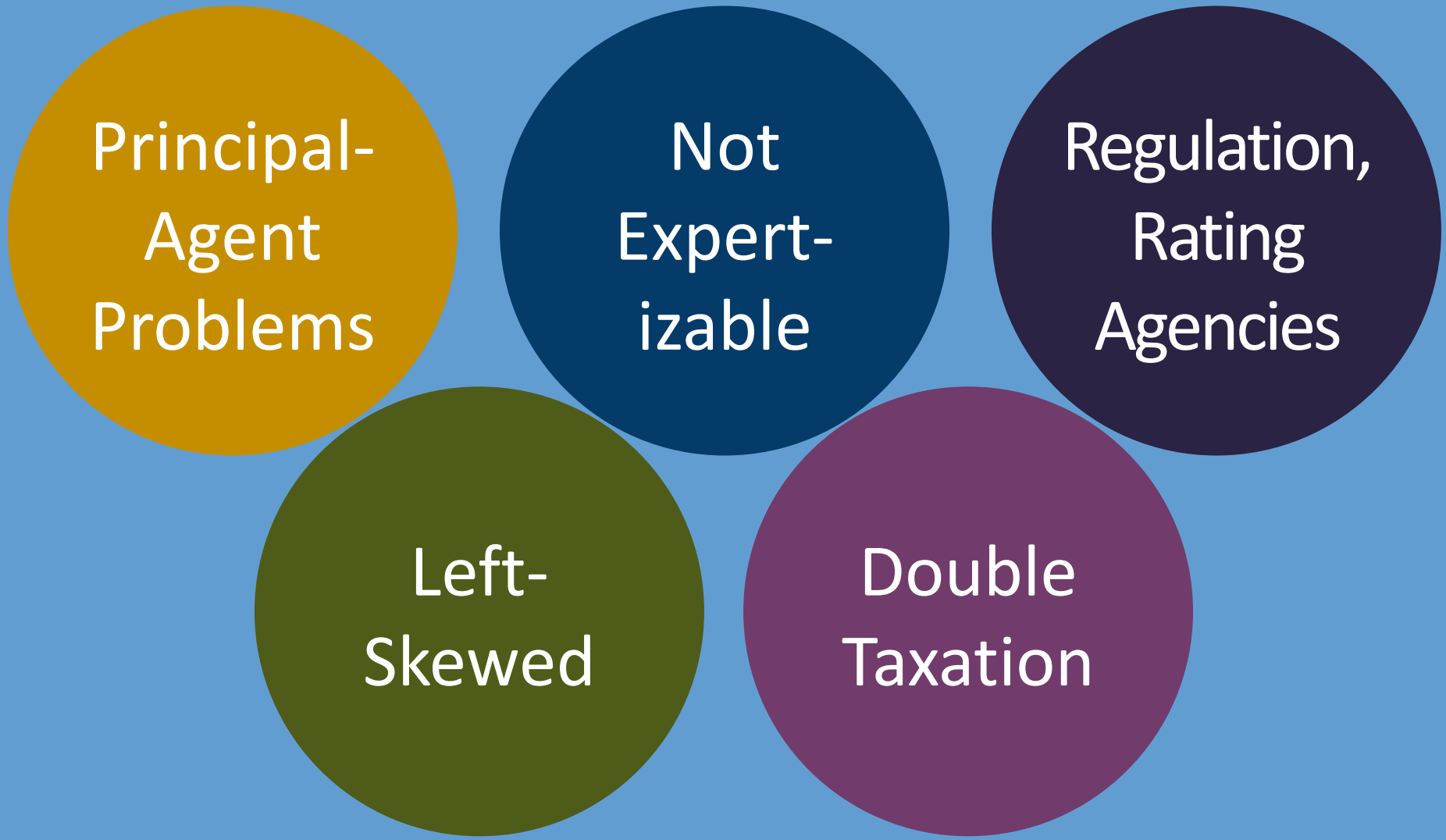
Classical All-Equity Pricing

$$P = EL + r(a - P)$$

Risk vs. ambiguity/uncertainty

Risk vs. ambiguity/uncertainty

Why Equity is Expensive

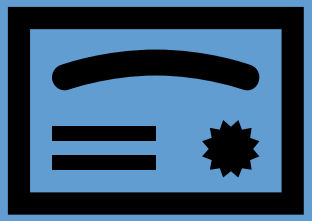


Capital vs. Equity

Capital = assets net of policyholder liabilities

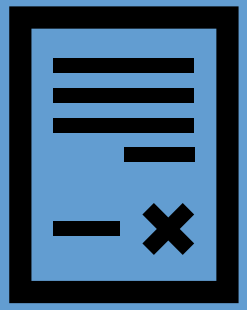
Equity = owner's residual value

Three Types of Capital



Equity

"Common" Quota share



Debt

Tranched Triggers default

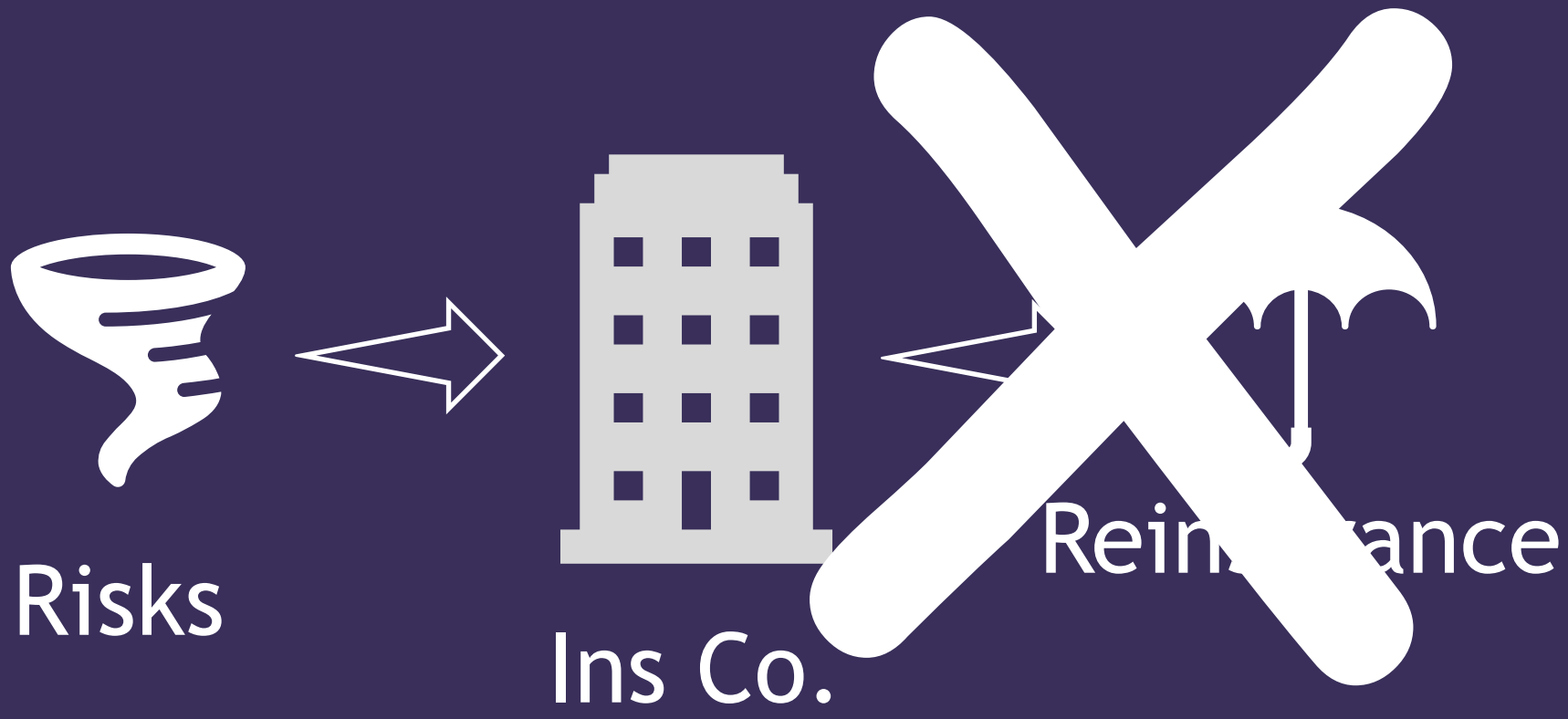
Reinsurance

Tranched Off-balance sheet

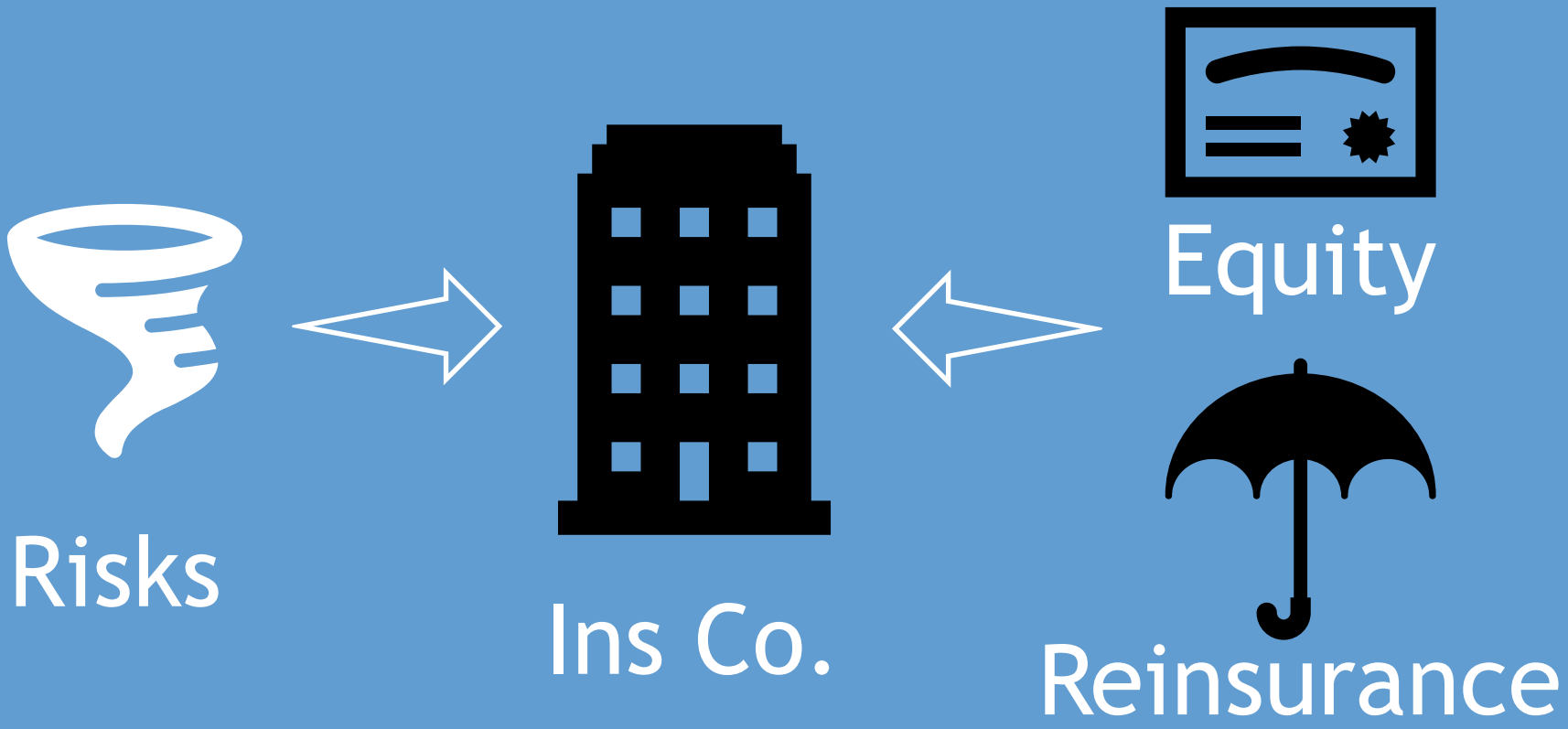
All-equity balance sheet pricing

All-equity balance sheet pricing

Reinsurance = Outwards Business



Reinsurance = Inwards Capital



Cat Bond Capital Can Be Cheaper

No Discretion

Expert-iz**able**

No Regulation

$\beta=0$

No Taxation

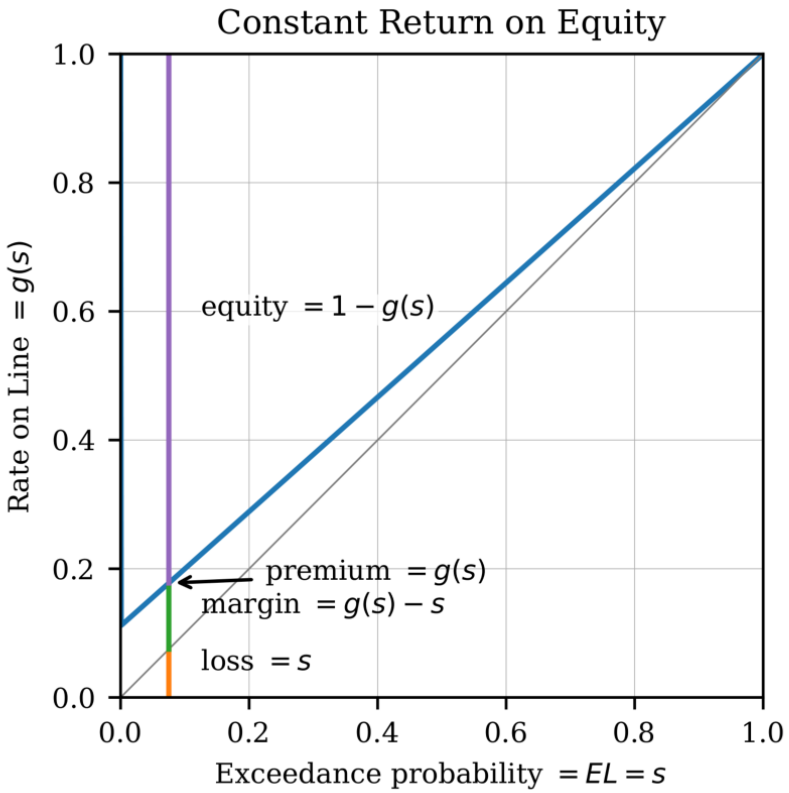
Live Poll

Which best describes your familiarity with distortion (or spectral) risk measures, such as the proportional hazards transform or the Wang transform?

- No familiarity
- Aware, but have never used
- Experimented, but do not use in “production”
- Implemented in day-to-day production pricing tools
- What? There are other ways to price?

Live Poll Answers

Fund Risk-Bearing Assets by Layer



$$P = vEL + da$$

$$P(x) = vS(x) + d$$

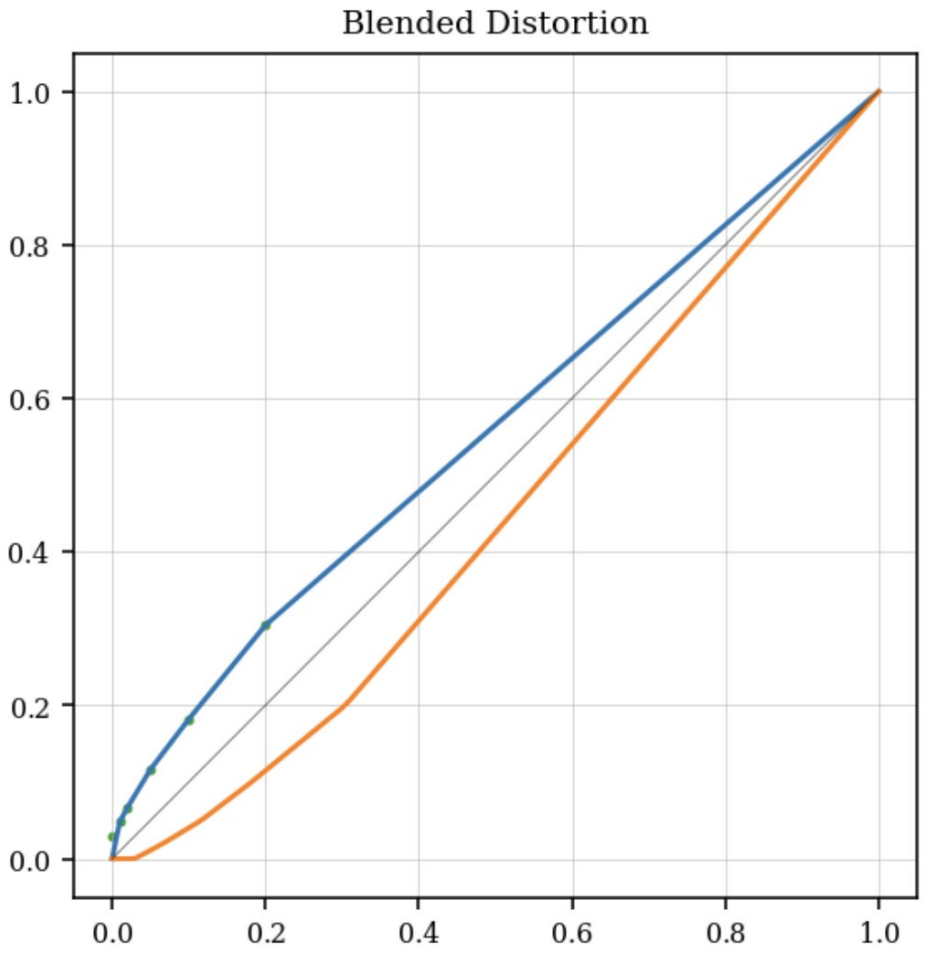
How to Calibrate Distortion g

{risk} × {capital structure} → price

Distortion

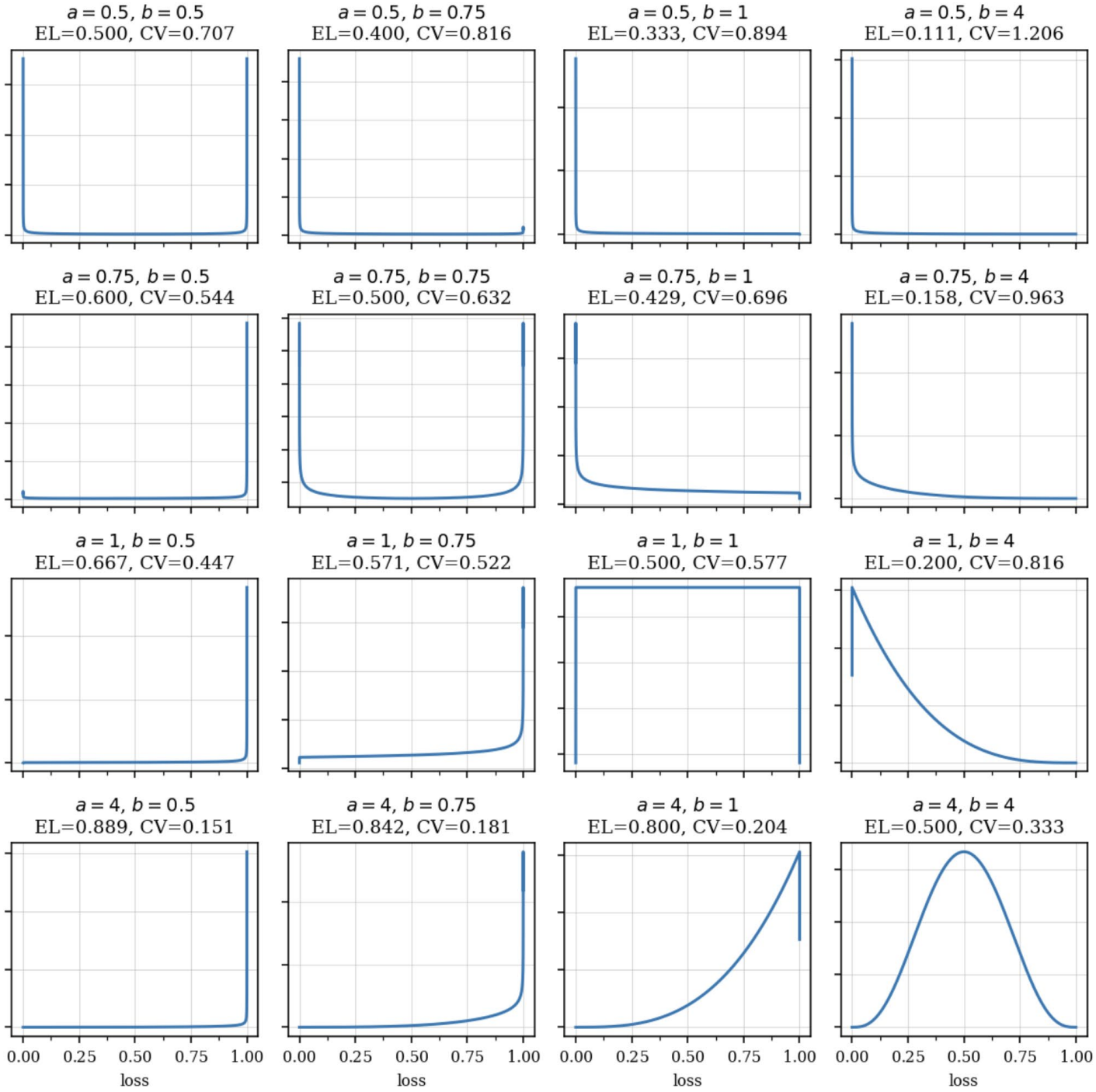
Equity + Reinsurance Distortion

s	ROE	$g(s)$
1e-10	0.03	0.0291
0.01	0.04	0.0481
0.02	0.05	0.0667
0.05	0.075	0.116
0.1	0.1	0.182
0.2	0.15	0.304

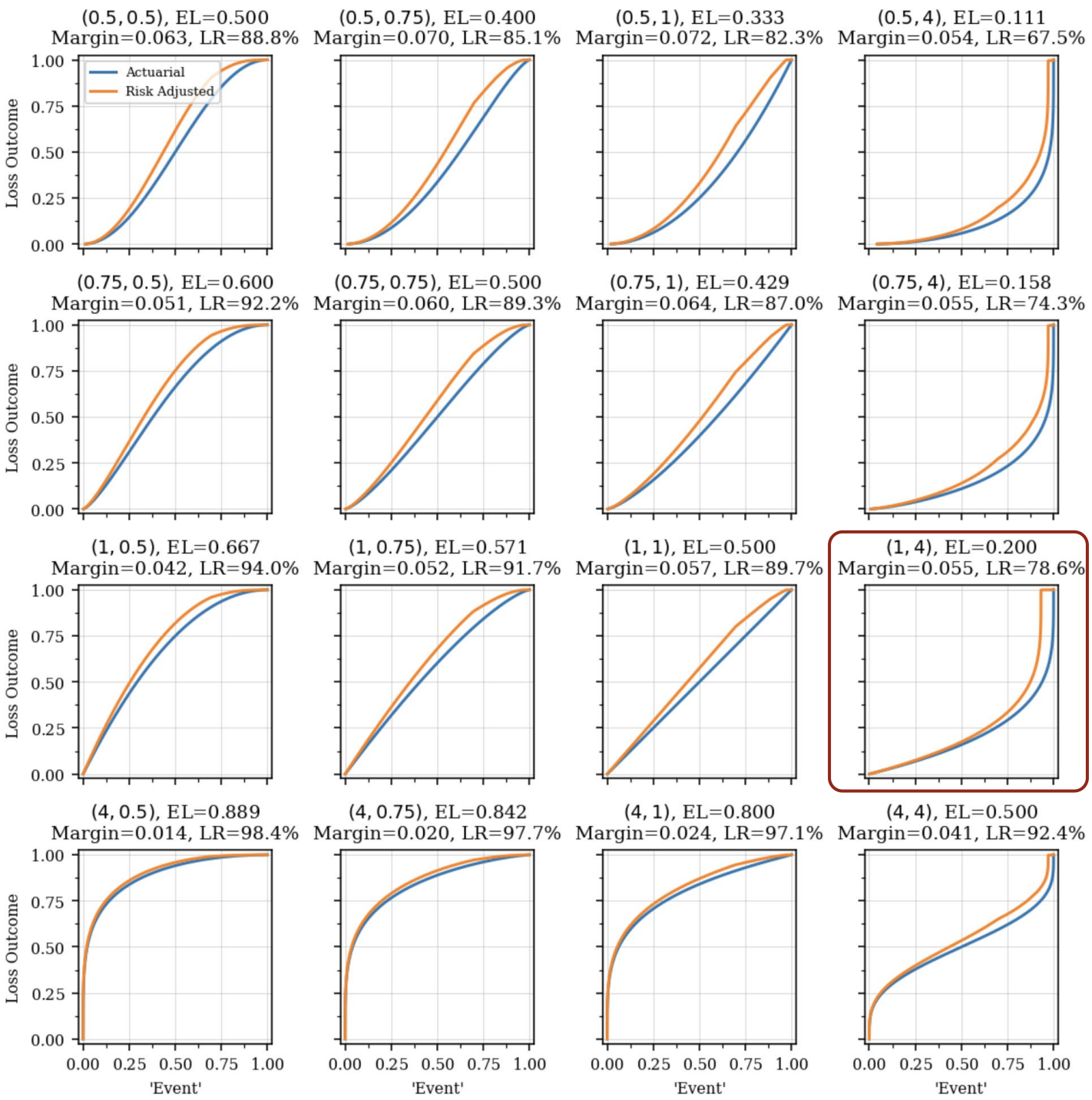


Portfolio Pricing

- $EL = E[X] = \int S(x)\,dx = \int xf(x)\,dx$
- $P = E[Xg'S(X)] = \int gS(x)\,dx = \int xg'S(x)f(x)\,dx$
- $g'S(X)$ is a risk adjustment, co-measure
- No default!

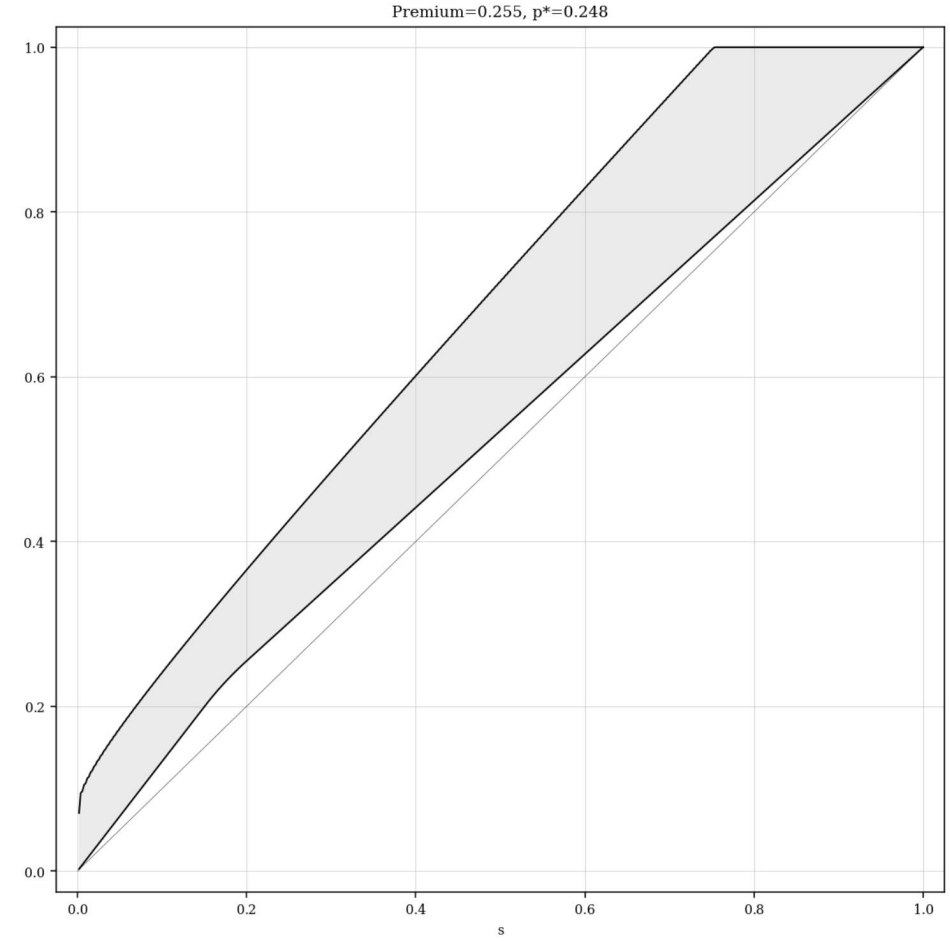


Beta Loss Densities



Loss, Margin, Loss Ratio

How to Calibrate Distortion g



Distortions replicating base pricing on (1,4)-beta distribution, 78.6% loss ratio

There is a unique lowest cost capital structure

Allocation When $X = \sum X_i$

- $EL_i = E[X_i] = \int x_i f(x)\, dx$
- $P_i = E[X_i g'S(X)] = \int x_i g'S(x) f(x)\, dx$
- Call P_i the **natural allocation**
- Terms and conditions apply

Allocation in Practice

Spreadsheet or SQL or R or Python or ...

- Rows = simulations
- Columns = loss by line and in total
- Group-by & sort total loss and average
- Compute $S(x)$ and $gS(x)$, difference
- Sum-product

Ambiguity

Ambiguity is not Bayes-able

- Tossing an uncertain coin
- Bayes solution → Law of Large Numbers
- Adverse selection → Wrong until the end of time

Ambiguity and Distortions

- Pr(H) > 0.5 and Pr(T) > 0.5
- Non-additive probabilities: $g(\Pr(A))$
- Example: for $g(s) = x^{1/2}$

  $g(\Pr(H)) = 0.71 = g(\Pr(T))$

  $g(\Pr(H, T)) = 1$

Underwriters and Distortions

- Distortion pricing = worst of a set of scenario outcomes

- The distortion controls how much event probabilities can increase

Live Poll

A legitimate pricing rule that is consistent with finance theory must have which of the following properties?

1. It only charges for non-diversifiable risk
2. It is linear: the price of a bundle is the sum of the prices of the parts
3. It does not allow arbitrage opportunities

Answers

- 1 only
- 2 only
- 3 only
- 1, 2, and 3
- Don’t understand the question

Live Poll Answers

Finance Theory and Distortions

- Bid-ask spread = market imperfection
- Decreases the effectiveness of no-arbitrage
- Distortion pricing is consistent with financial theory

Optimization and Distortions

- Optimization = marginal improvements

- Natural allocation has a marginal interpretation

$$\lim_{t \downarrow 0} \frac{\rho(X + tX_i) - \rho(X)}{t} = E[X_i g'S(X)]$$

Top Ten Reasons To Love Distortions

1. Calibrate to capital structure
2. Can calibrate to market pricing
3. Practical spreadsheet implementation
4. Easy to work with catastrophe model output
5. Sensitive to shape of risk
6. Marginal risk interpretation
7. Weighted average of TVaR interpretation
8. Worst-over-scenarios interpretation
9. Consistent with underwriting
10. Consistent with financial theory

Part II: Reinsurance Applications

SRM Use Cases

Policy pricing / technical premium

LOB assessment

LOB capital cost allocation

Reinsurance decisions

SRM Principal Modes

- "A/B": Evaluate $\rho(X_G)$ and $\rho(X_N)$; compare

- Allocate: $NA(X_C; X_G)$ where $X_C = X_G - X_N$

Applying the Modes

Use Case	A/B	Alloc
Policy pricing / technical premium	No	Yes
LOB assessment	??	Yes
LOB capital cost allocation	No	Yes
Reinsurance decisions	Yes	Yes

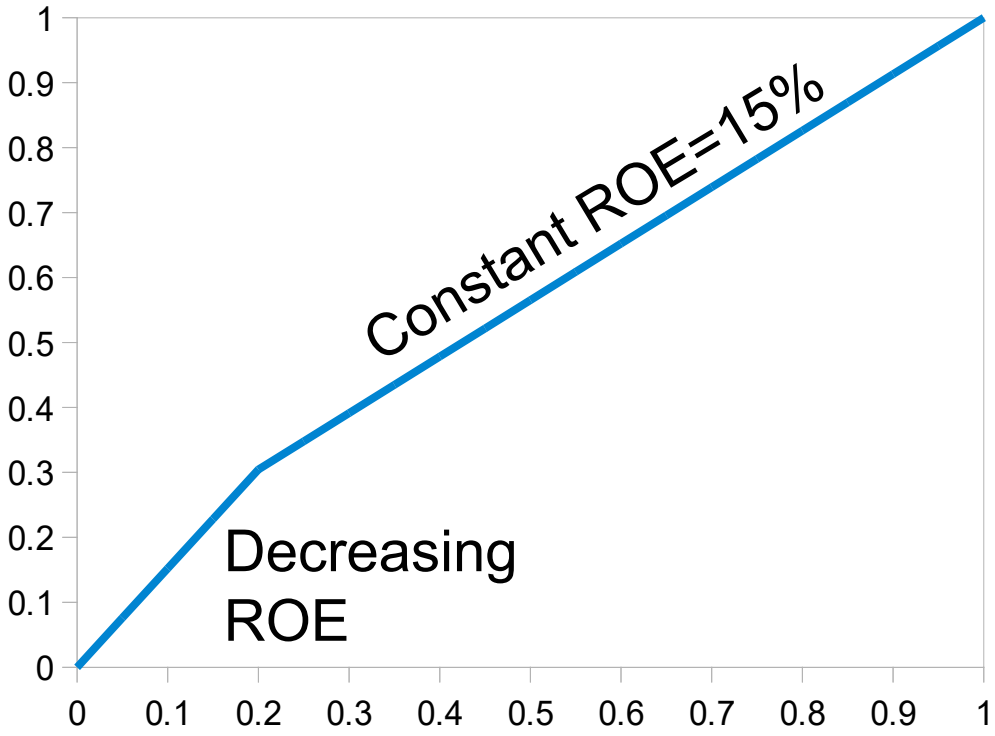
Case Study

- Use reinsurance to minimize total cost of risk
- VERY simple portfolio: 2 lines
- VERY simple XOL contract: on line 1
- Illustrate the two modes

Reinsurance Example

Gross				Net 2 xs 2 (X1), premium = 0.38			
X1	X2	X_G	p	$X1_C$	$X1_N$	X2	X_N
1	1	2	0.7	0	1	1	2
2	3	5	0.1	0	2	3	5
4	2	6	0.1	2	2	2	4
3	4	7	0.1	1	2	4	6

Our Distortion Function



First Mode: A/B

- A/B the gross and net positions

- Which one has lower total cost?

Gross Valuation

X_G	X_N	p	S	g(S)	dg_G
2	2	0.7	0.3	0.391	0.609
5	5	0.1	0.2	0.304	0.087
6	4	0.1	0.1	0.152	0.152
7	6	0.1	0	0	0.152

$$\sum X_{Gi}\, dg_i = 3.630$$

= gross premium

Net Valuation

X_G	X_N	p	S	g(S)	dg_N
2	2	0.7	0.3	0.391	0.609
6	4	0.1	0.2	0.304	0.087
5	5	0.1	0.1	0.152	0.152
7	6	0.1	0	0	0.152

Re-order!

$$\sum X_{Ni}\, dg_i = 3.239$$

= net premium

A/B Gross and Net

3.239	Net premium
+0.380	Cost of reinsurance
=3.619	Total cost, net position

vs

3.630	Gross premium + 0 r/i cost
0.011	Difference

Conclusion: r/i a marginally good deal.

Second Mode: Allocation

- Allocate premium from Gross to Net and Ceded components
- Allocate premium from Net to Gross and (-)Ceded components
- Is Ceded worth the r/i premium?

Allocating Ceded from Gross Position

$$X_G = X_N + X1_C$$

X1	X2	X_G	dg_G	$X1_C$
1	1	2	0.609	0
2	3	5	0.087	0
4	2	6	0.152	2
3	4	7	0.152	1

$$\Sigma\ X1c_i\ dg_i = 0.456 > 0.38 \text{ by } 0.076$$

Allocating Ceded from Net Position

$$X_N = X_G - X1_C$$

$X1_N$	X2	X_N	dg_N	$X1_C$
1	1	2	0.609	0
2	2	4	0.087	2
2	3	5	0.152	0
2	4	6	0.152	1

Re-order!

$$\sum X1c_i\, dg_i = 0.326 < 0.38 \text{ by } 0.054$$

Conclusions about the r/i deal so far

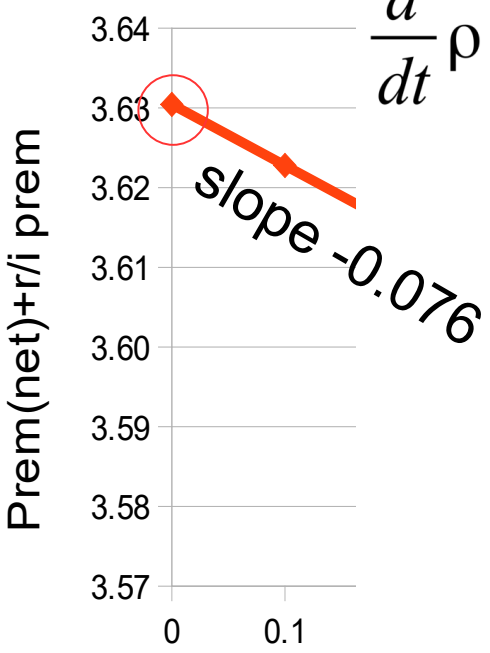
- Direct A/B:
  - Eh, okay. +0.011
- Allocate starting at gross
  - Great! +0.076
- Allocate starting at net
  - Bad! -0.054

- How do I explain this to my boss?
  - What does it all really mean?

A More Thorough A/B

- Fractional participation t
- $X_N(t) = X_G - t \cdot X_C$
- Minimize total cost = $\rho(X_N(t)) + t \cdot$ (r/i prem)

Starting at Gross



$$\frac{d}{dt}\rho\left(X_G - t\cdot X_C + t\cdot\pi\right) = -E\left[dg_G\cdot X_C\right]+\pi$$

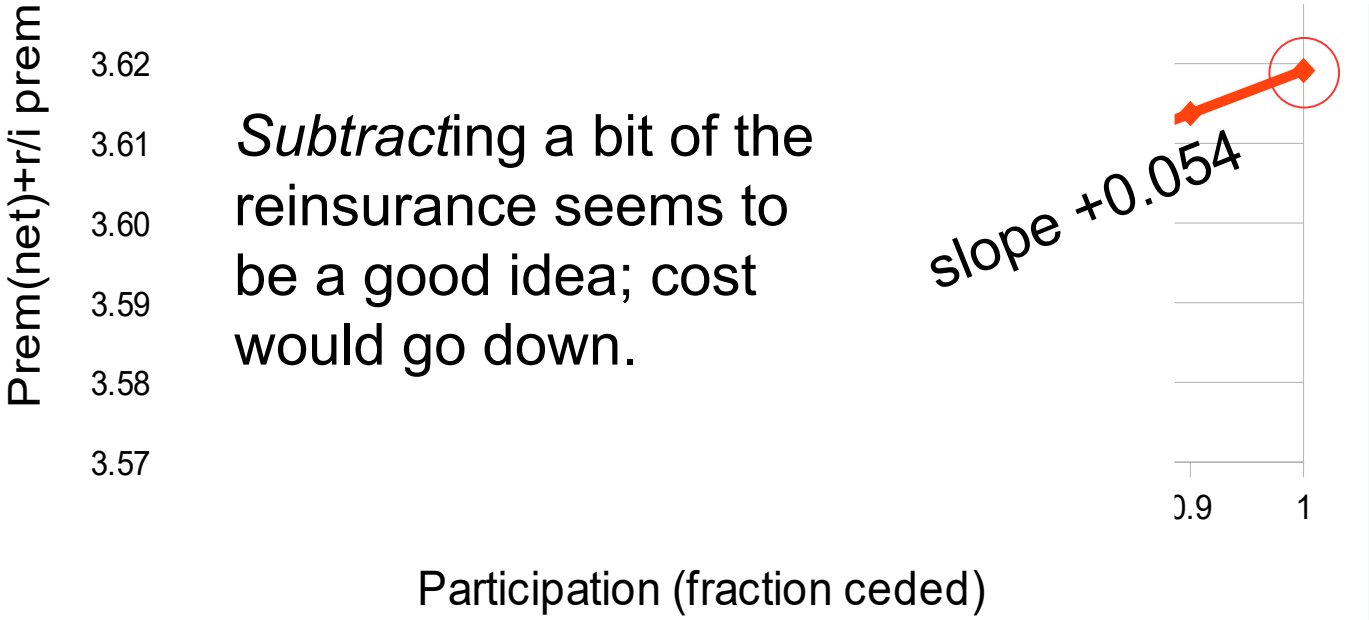
Allocation is the gradient!

Adding a bit of the reinsurance seems to be a good idea; cost is going downward

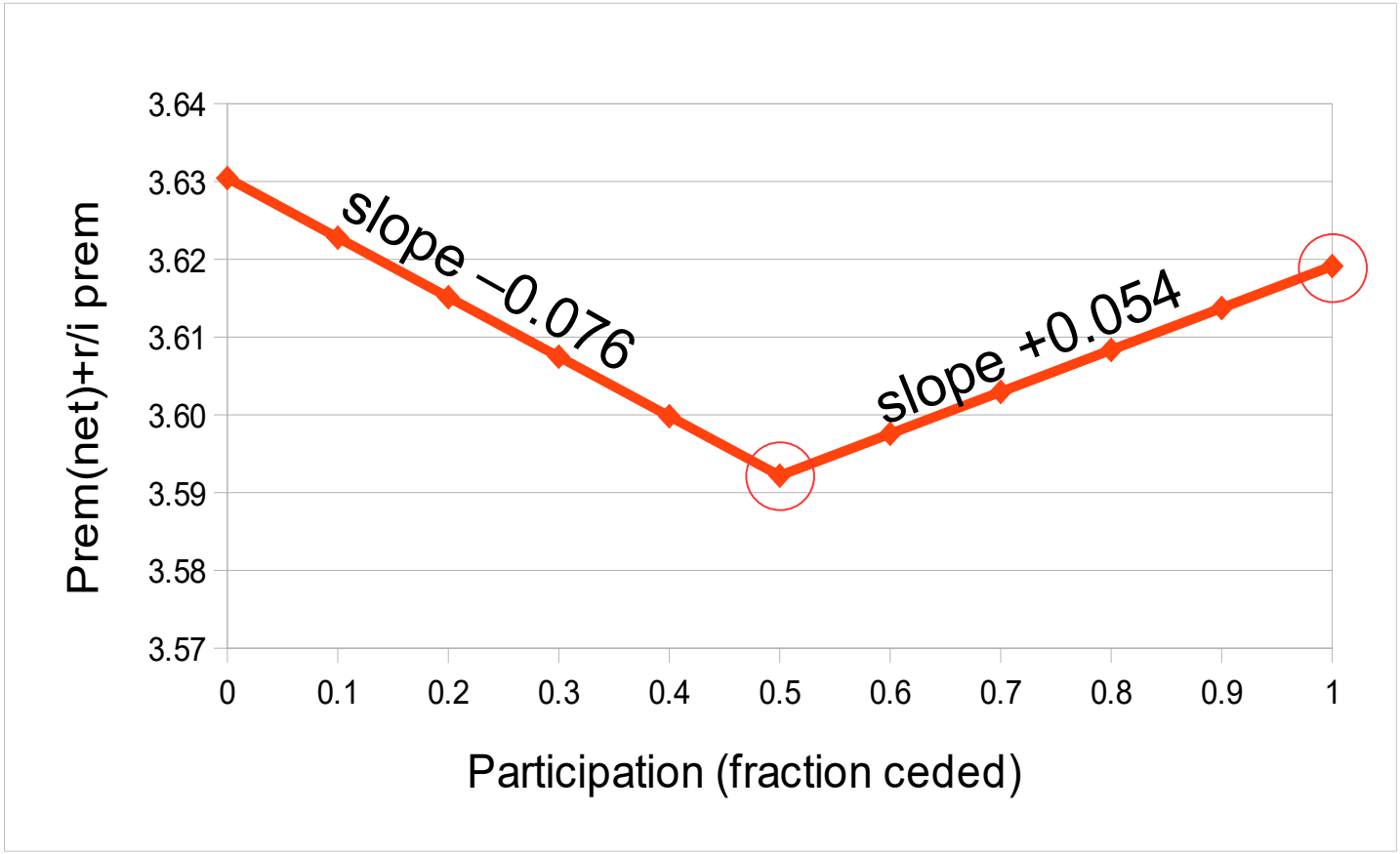
Starting at Net

$$\frac{d}{dt}\rho\left(X_N+(1-t)\cdot X_C+t\cdot\pi\right)=-E\left[dg_N\cdot X_C\right]+\pi$$

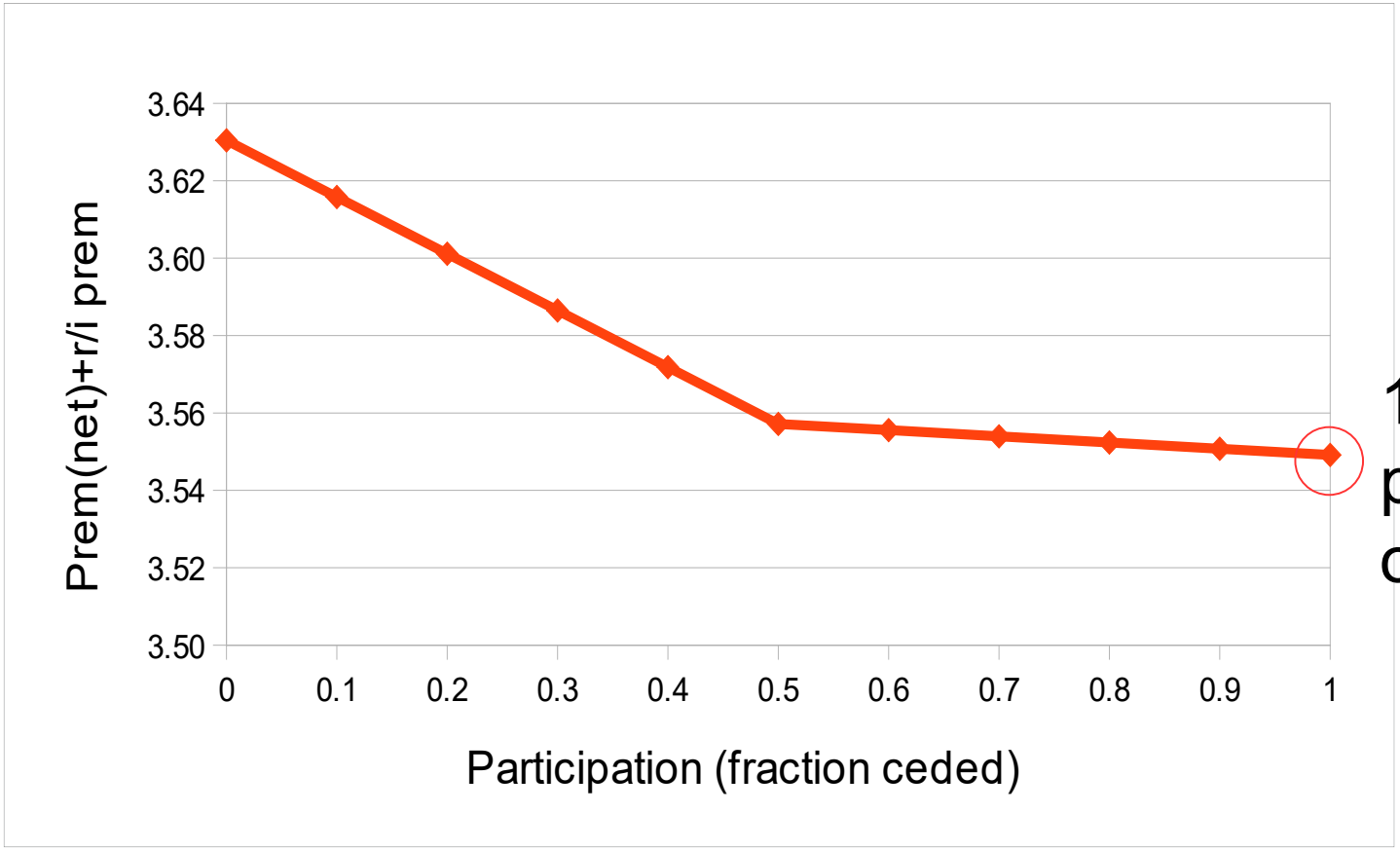
*Subtract*ing a bit of the reinsurance seems to be a good idea; cost would go down.



The Full Story

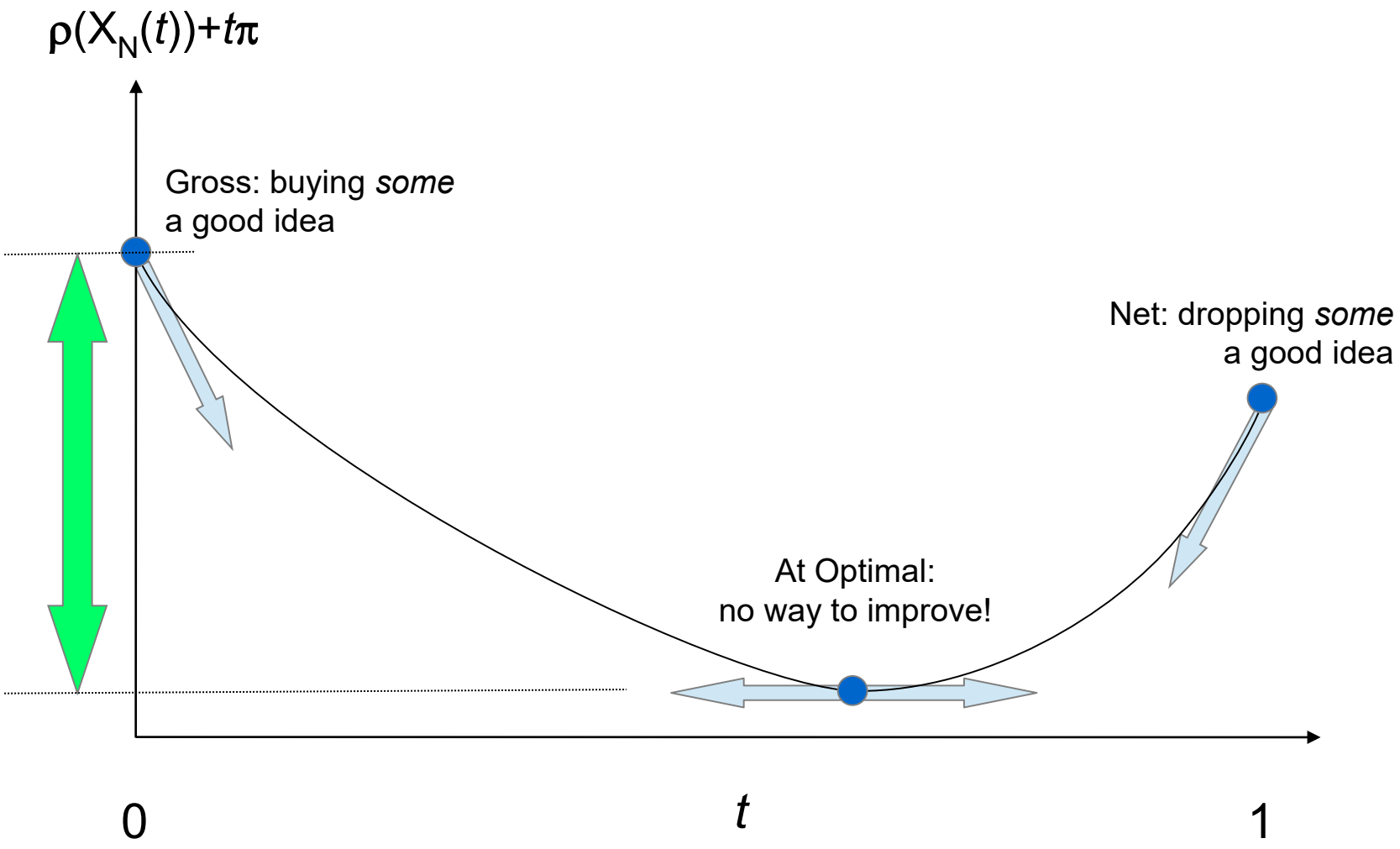


If Premium Were 0.31



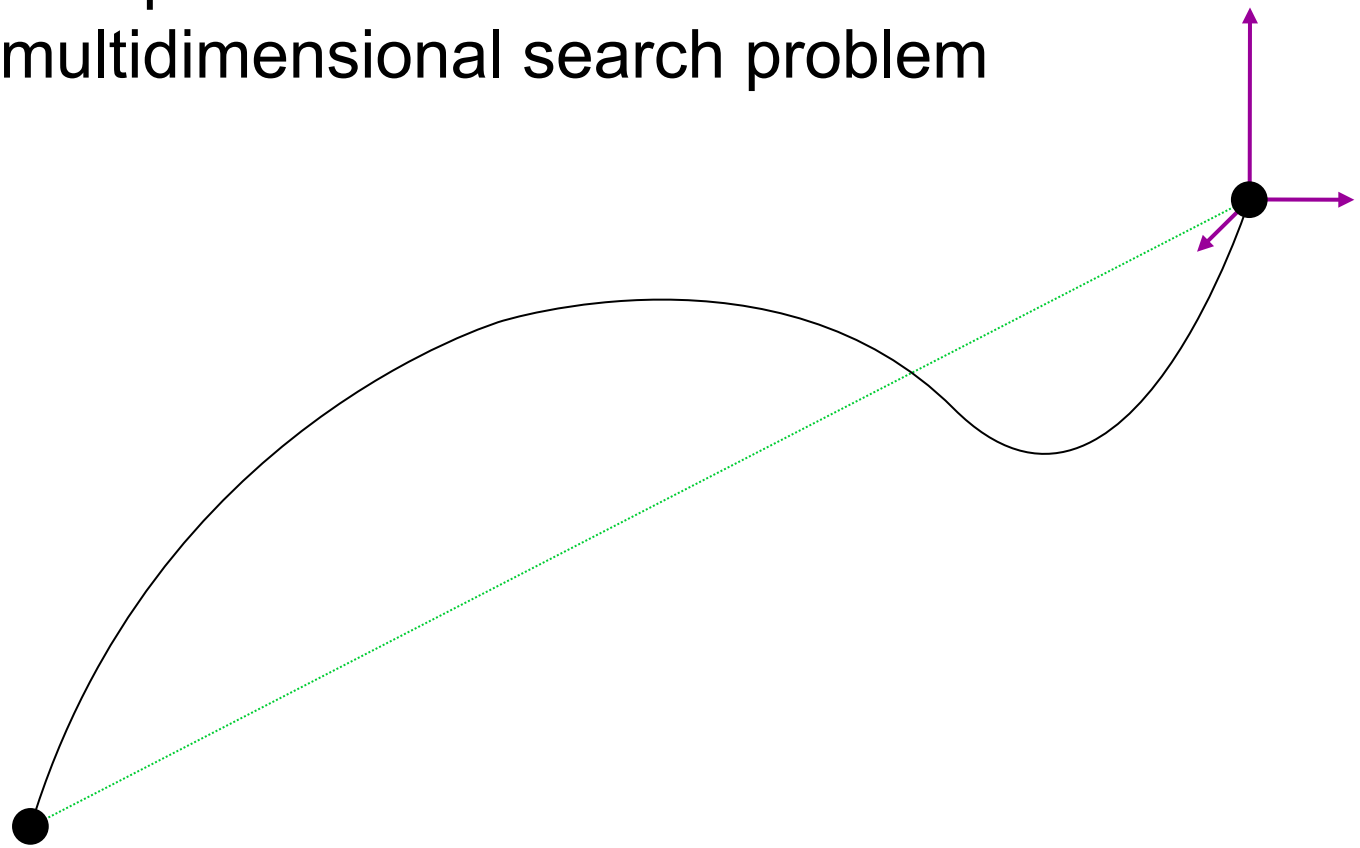
100% placement optimal

More generally, curved



Advanced Application

Multiple contracts:
multidimensional search problem



Summary: Allocation for Decisions

- Allocation = gradient
- Faster than A/B
- Like using Taylor's Theorem
  - OK for small changes
  - Iffy for big

Applying the Modes, Redux

Use Case	A/B	Alloc
Policy pricing / technical premium	No	Yes
LOB assessment	??	Yes
LOB capital cost allocation	No	Yes
Reinsurance decisions	Yes	Yes

Questions?